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A SIMPLE STOCHASTIC MODEL OF TWO PHASE FLOW PRESSURE DROP ACCOMPANIED BY BOILING INSIDE CIRCULAR TUBES WITH IN-LINE STATIC MIXERS

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Abstract—A simple stochastic model has been developed for boiling pressure drop inside a circular tube with in-line static mixers. This model gives rise to the dimensionless correlating equation of the form:

$$[f] = a[Pr_m]^{-1/2}[Re_L]^{-1}\left[\frac{\mu_m}{\bar{\mu}_L}\right]\left[\frac{\rho_m}{\bar{\rho}_b}\right]^{-1/2}\left[\frac{H_{st} + xH_{LG}}{C_{pm}\Delta T}\right]^{-1/2}$$

This correlation shows good agreement with the experimental data.

INTRODUCTION

The purpose of this study is to develop a simple stochastic model for two phase flow pressure drop inside a circular tube with in-line static mixers accompanied by boiling.

In 1950, Einstein & Li (1950) proposed a simple stochastic model for momentum transfer between the fluid particles and the tube wall in a turbulent condition. Meek & Baer (1970) extended the model by removing some of the assumptions. These successful applications of the simple stochastic model to momentum transfer inspired one to examine the applicability of this model to the two phase pressure drop accompanied by boiling inside circular tubes with in-line static mixers.

Experimental evidence (Azer *et al.* 1980) indicates that in-line static mixers are highly effective in enhancing the rate of boiling heat transfer inside a circular tube with only a moderate increase in pressure drop across the tube. The static mixers are constructed of a number of short elements of right- and left-hand helices (Chen 1975; Genetti & Priebe 1973; Nauman 1979) (also see figure 1). These elements are orientated so that each leading edge is at

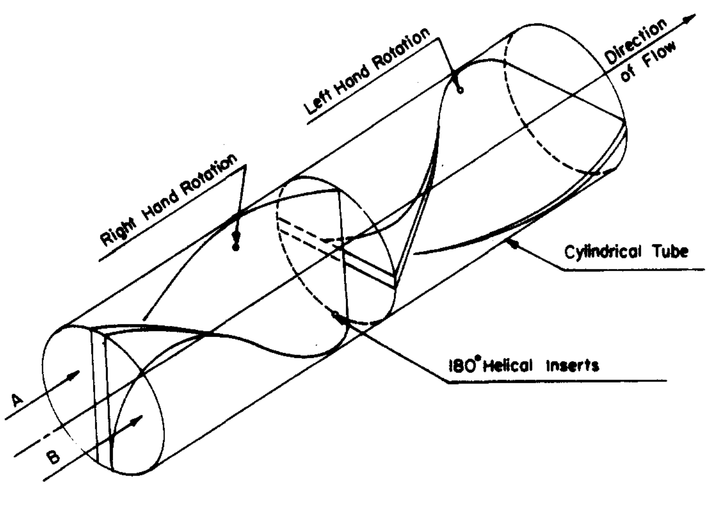

Figure 1. Schematics of Kenics static mixing unit.

90° to the trailing edge of the one ahead. Because of the geometric features of the mixers, the flow stream divides at the leading edge of each element and follows the semi-circular channel created by the element's shape. At each succeeding element, the two flows are further divided, resulting in an exponential progression of flow divisions. This sequence of flow division and rotation gives rise to thorough and efficient radial mixing.

The unique characteristics of in-line static mixers and the boiling phenomenon suggest that the simple stochastic approach may be suitable for modeling the two phase pressure drop accompanied by boiling inside a circular tube with in-line static mixers.

FORMULATION

The flow pattern in a boiler tube with in-line static mixers has been observed to be homogeneous with liquid and bubbles well mixed and vigorously agitated throughout the cross-section of the tube (Azer *et al.* 1980). Assuming that momentum is transferred from a packet of the bubble-liquid mixture to the inner tube surface by shear force during the contact between them, one can write

$$\frac{\partial u}{\partial \theta} = \frac{\mu_m}{\rho_m}\frac{\partial^2 u}{\partial y^2} = \nu_m \frac{\partial^2 u}{\partial y^2} \qquad [1]$$

where u denotes the velocity, θ is the contact time, μ_m is the mean dynamic viscosity of the bubble–liquid mixture, ρ_m is the mean density of the bubble–liquid mixture, y is the distance away from the tube wall, and ν_m is the mean kinematic viscosity of the bubble–liquid mixture. The solution of [1] subject to appropriate initial and boundary conditions can be expreseed as (Fan *et al.* 1978)

$$\frac{u - u_b}{-u_b} = \left[1 - \frac{y}{\delta} - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\left(\sin\frac{n\pi y}{\delta}\right)e^{(-\pi^2 n^2 \nu_m \theta/\delta^2)}\right] \qquad [2]$$

where u_b denotes the mean velocity of the bubbles, and δ is the size or thickness of the bubble–liquid packets. The instantaneous rate of momentum transfer at the wall at a contact time θ, $\sigma_i(\theta)$, and the average rate of momentum transfer, σ, can be written, respectively, as

$$\sigma_i(\theta) = \mu_m \frac{\partial u}{\partial y}\bigg|_{y=0} \qquad [3]$$

and

$$\sigma = \int_0^\infty \sigma_i(\theta)\phi(\theta)\,d\theta \qquad [4]$$

where $\phi(\theta)$ is the contact time or waiting period (Hsu & Graham 1976) distribution function of the bubble–liquid packets. By considering that the bubble–liquid packets on the surface are replaced in a completely random manner (Danckwerts 1951; Toor & Marchello 1958), because of vigorous agitation of the bubble–liquid mixture in the boiler tube (Azer *et al.* 1980) one has

$$\phi(\theta) = s\,e^{-s\theta} \qquad [5]$$

where s is the average frequency of surface renewal (Fan *et al.* 1978). If one assumes that the mean period of contact between the bubble–liquid packet and the boiler tube surface, $\bar{\tau}$, is proportional to the mean residence time of the bubble–liquid mixture in the boiler tube, $\bar{t}$, the average frequency of the surface renewal, s or $1/\bar{\tau}$, can be related as

$$s \equiv \frac{1}{\bar{\tau}} \propto \frac{1}{\bar{t}} \quad [6]$$

Substitution of [2] into [3] and substitution of the resultant expression and [5] into [4] give the average rate of momentum transfer across a unit area of the tube surface or wall as

$$\sigma = u_m \cdot (s\mu_m\rho_m)^{1/2} \coth\left(\frac{s\delta^2}{\nu_m}\right)^{1/2} \quad [7]$$

where u_m is the mean velocity of the bubble–liquid mixture. For sufficiently large δ, s, or $1/\nu_m$, one has

$$\coth\left(\frac{s\delta^2}{\nu_m}\right)^{1/2} \simeq 1. \quad [8]$$

Then, because of [8] and [6], [7] reduces to

$$\sigma = u_m \cdot \left(\frac{\nu_m\rho_m^2}{\bar{\tau}}\right)^{1/2} = u_m\rho_m\sqrt{\frac{\nu_m}{\bar{\tau}}} \propto u_m\rho_m\frac{\nu_m}{\bar{t}} \quad [9]$$

By definition, the mean residence time, $\bar{t}$, in this expression is

$$\bar{t} \equiv \frac{L}{u_m} \quad [10]$$

where L is the length of the tube. The mean velocity of the bubble–liquid mixture, u_m, can be derived as follows:

For a circular boiler tube, with an inside diameter of d and a length of L, the volumetric flow rate of the bubble, $\dot{V}_b$, can be written as

$$\dot{V}_b = \bar{u}_b \cdot A_b \quad [11]$$

where $\bar{u}_b$ is the mean velocity of the bubbles, and A_b, the equivalent cross-sectional area of the bubble flow. Since the bubbles and liquid are well mixed and agitated vigorously at any position along the tube, $\bar{u}_b$ and A_b can be written, in general, as

$$\bar{u}_b \simeq u_m \quad [12]$$

$$A_b \propto \delta_{ave} d \quad [13]$$

the δ_{ave} is the average size or thickness of the packets of the bubble–liquid mixture. Note that d and δ_{ave} are assumed to be essentially independent of the operating conditions. Substituting [12] and [13] into [11], and solving the resultant expression for the mean velocity of the bubble–liquid mixture, u_m, one obtains

$$u_m \propto \frac{\dot{V}_b}{\delta_{ave} d}. \quad [14]$$

Since the heat supplied to boil the liquid is equal to the sum of the sensible and latent heats, one has from the heat balance at any position along the tube

$$Q = -k_m A \left.\frac{\partial T}{\partial y}\right|_{y=0} = (H_{se} + xH_{LG}) \cdot \dot{M} \quad [15]$$

where Q denotes the total heat transfer rate, k_m is the mean thermal conductivity of the mixture, A is the heat transfer area, T is the temperature, H_{se} is the sensible heat of heating liquid, x is the dryness fraction of heating liquid at the exit, H_{LG} is the latent heat of boiling, and $\dot{M}$ is the mass flow rate of the boiling fluid. Furthermore, one may write

$$\begin{aligned} -k_m A \left.\frac{\partial T}{\partial y}\right|_{y=0} &\simeq -k_m A \left.\frac{\Delta T}{\delta_{\text{ave}}}\right|_{y=0} \\ &\simeq (H_{se} + xH_{LG})\, \dot{M} \end{aligned} \quad [16]$$

where ΔT is the temperature difference between the bulk and the tube wall. Solving [16] for δ_{ave}, one obtains

$$\delta_{\text{ave}} \simeq \frac{k_m A \Delta T}{(H_{se} + xH_{LG})\, \dot{M}}. \quad [17]$$

Substitution of [17] into [14] yields

$$u_m \propto \frac{\dot{V}_b (H_{se} + xH_{LG}) \dot{M}}{d k_m A \Delta T} \quad [18]$$

where

$$A = \pi L D \text{ and } \dot{V}_b \bar{\rho}_b \propto \dot{M}$$

in which D is the diameter of the tube and $\bar{\rho}_b$ is the mean density of the bubble. The Fanning friction factor for a homogeneous flow, f, can be defined as (Knudsen & Katz 1958)

$$f \equiv \frac{g_c(-\Delta P) d}{2 u_m^2 \rho_m L} = \frac{2\sigma g_c}{u_m^2 \rho_m} \quad [19]$$

where ΔP denotes the pressure difference and g_c is the proportionality constant. Substitution of [10] and [18] into [9] and substitution of the resultant expression into [19] give the Fanning friction factor as

$$f \propto \left[\frac{d^2 \mu_m k_m \bar{\rho}_b \Delta T}{\rho_m \dot{M}^2 (H_{se} + xH_{LG})}\right]^{1/2}. \quad [20]$$

In dimensionless form, this expression becomes

$$f = a\left[\frac{k_m}{\mu_m C_{pm}} \frac{d^2 \bar{\mu}_L^2}{\dot{M}^2} \frac{\mu_m^2}{\bar{\mu}_L^2} \frac{\bar{\rho}_b}{\rho_m} \frac{C_{pm}\Delta T}{(H_{se} + xH_{LG})}\right]^{1/2}$$

or

$$[f] = a[\mathrm{Pr}_m]^{-1/2} [\mathrm{Re}_L]^{-1} \left[\frac{\mu_m}{\bar{\mu}_L}\right] \left[\frac{\rho_m}{\bar{\rho}_b}\right]^{-1/2} \left[\frac{H_{se} + xH_{LG}}{C_{pm}\Delta T}\right]^{-1/2} \quad [21]$$

where C_{pm} is the mean specific heat of the bubble–liquid mixture, $\bar{\mu}_L$ is the mean dynamic viscosity of the bubbling fluid, Pr_m denotes the Prandtl number of the bubble–liquid mixture, Re_L denotes the Reynolds number of the boiling fluid, and a is a proportionality constant to be recovered by experimental data.

VERIFICATION OF THE MODEL

Fifteen experimental data previously obtained (Azer *et al.* 1980) were used here to verify the model. The facilities and procedure employed in obtaining the data are outlined below.

The facilities included a liquid receiver, circulating pump, preheater, observation section and condenser. The facilities also included two identical test boiler tubes which were vertically mounted in parallel. One of the test tubes was a smooth tube and the other contained 50 static mixer elements (25 units). Each boiler tube was electrically heated and instrumented to measure the outside wall temperature at five equidistant axial locations as well as the inlet and outlet temperatures of refrigerant-113. The pressure drop across the test section was measured by a differential pressure transducer of type Pace-KP15. The adjustments of the electrical power and the opening of the by-pass valve of the circulating pump were used to change the mass flow rate of the refrigerant-113, the boiling temperature and pressure, and the heat flux to the test section. The electrical heat flux input and the refrigerant flow rate were the main parameters that were varied. The data recorded were the boiling temperature (53.7–54.2°C), the boiling pressure ($1.25\times10^5-1.26\times10^5\ \mathrm{N/m^2}$), the inlet subcooling ($18^\circ\sim20^\circ$), the mass velocity of refrigerant-113 (1.18×10^5–$3.82\times10^5\ \mathrm{kg/hr\cdot m^2}$), the heat flux ($1.69\times10^3\sim8.63\times10^3\ \mathrm{W/m^2}$) and the dryness fraction ($0.008\sim0.03$).

The pressure drop data obtained are plotted in figure 2, and the model, [21], is fitted to the data. A linear regression analysis has yielded $a = 1175.8$, with a correlation coefficient, r, of 0.94, and a standard deviation, S_{yx}, of 0.009. A test of linearity of the data by an analysis of variance involving the F-test (Graybill 1961), and a test of the null hypothesis, that the regression line passes through the origin by the t-test (Snedecor & Cochran 1967), have indicated that [21] adequately correlates the data.

CONCLUSION

A simple stochastic model of momentum transfer has been proposed to describe the forced flow boiling pressure drop inside a circular tube with in-line static mixers. A dimensionless

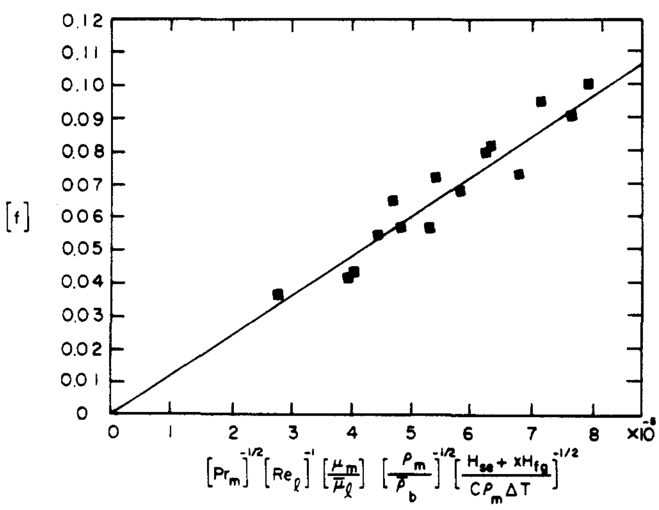

Figure 2. Fitting of the model to the experimental data.

correlation has been developed based on the model. It includes a correlation constant which has been determined from experimental measurements. Additional data, however, are needed to verify the general applicability of the correlation.

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